University of Anbar
College of Engineering
Civil Engineering Department

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Strength of Materials II, 2022 **Lecturer: Dr. Dhafer K. Jadaan**

Syllabus

1-Flexural stresses in beams (4 Weeks)

2) Transverse shear stresses (4 Weeks)

3) Stress and strain transformations (3 Weeks)

4) Deflection in beams (3 Weeks)

5) Buckling of Columns (2 Weeks)

Chapter One

Flexural stresses in Beams

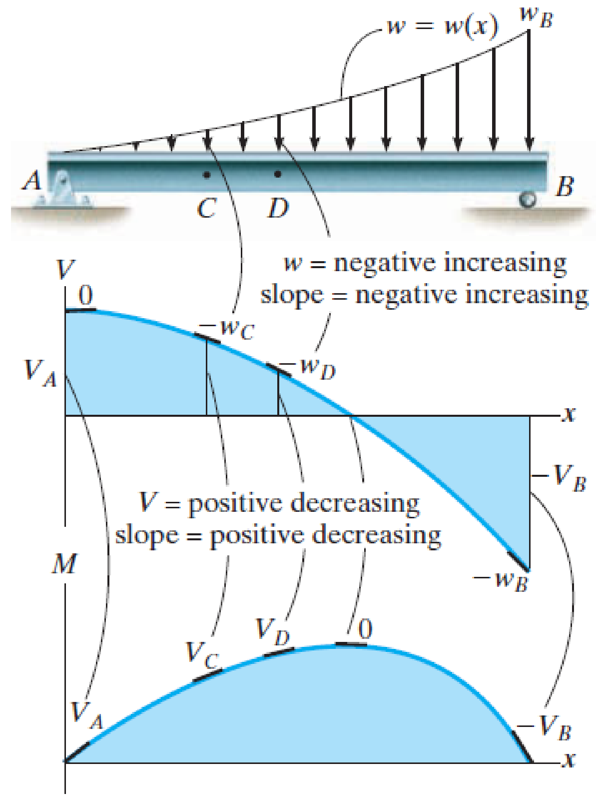
Review of Shear and Bending Moment Diagrams

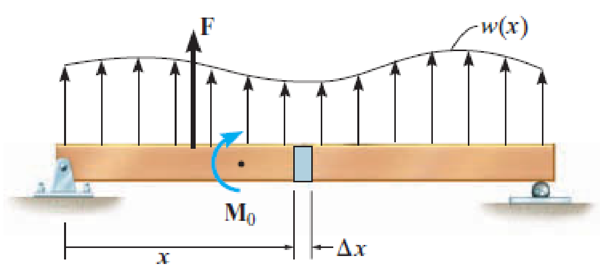
$$\frac{dV}{dx} = w(x)$$

slope of shear diagram at each point = distributed load intensity at each point

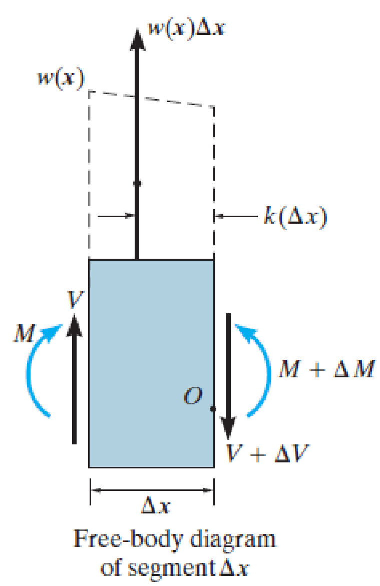
$$\frac{dM}{dx} = V$$

slope of moment diagram at each point = shear at each point





(a)



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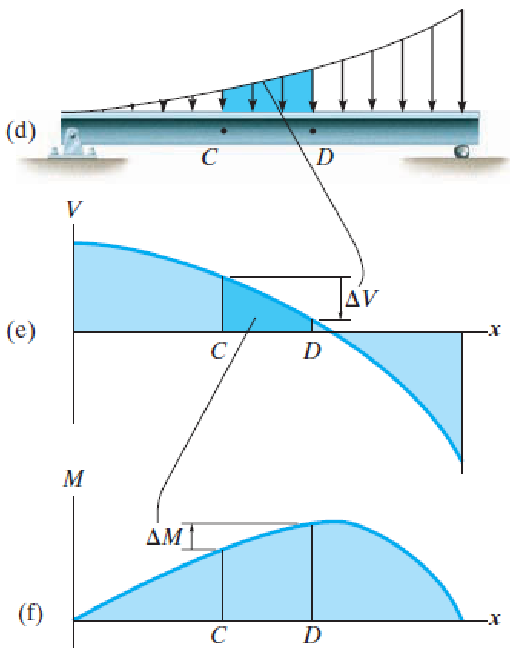


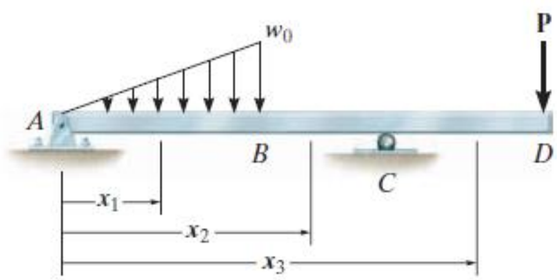
$$\Delta V = \int w(x)\, dx$$

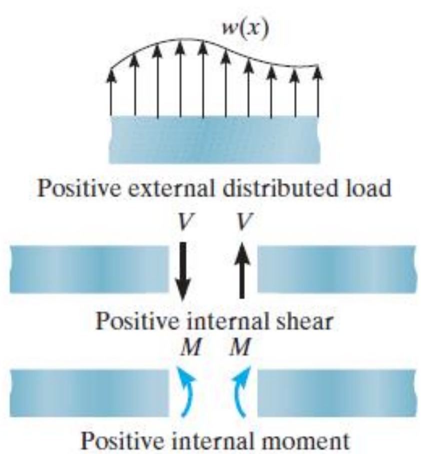
change in shear = area under distributed loading

$$\Delta M = \int V(x)\, dx$$

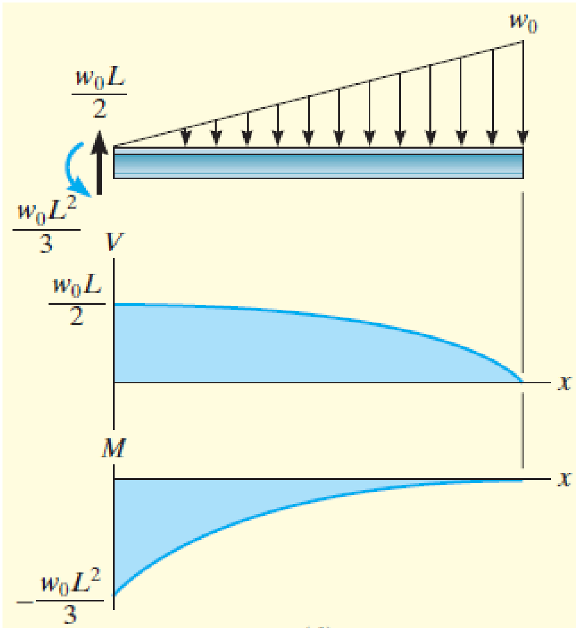
change in moment = area under shear diagram

Due to load changing, shear and bending moment are discontinuous and that is why they have to be calculated at different regions (x1, x2, x3,…)

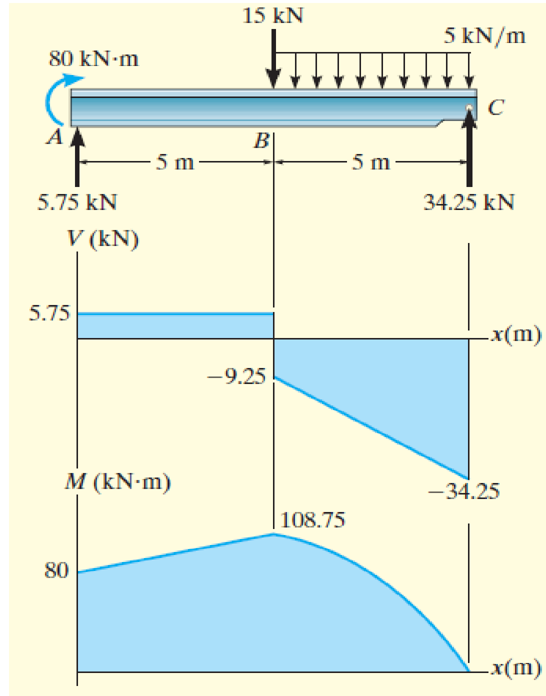




Beam sign convention

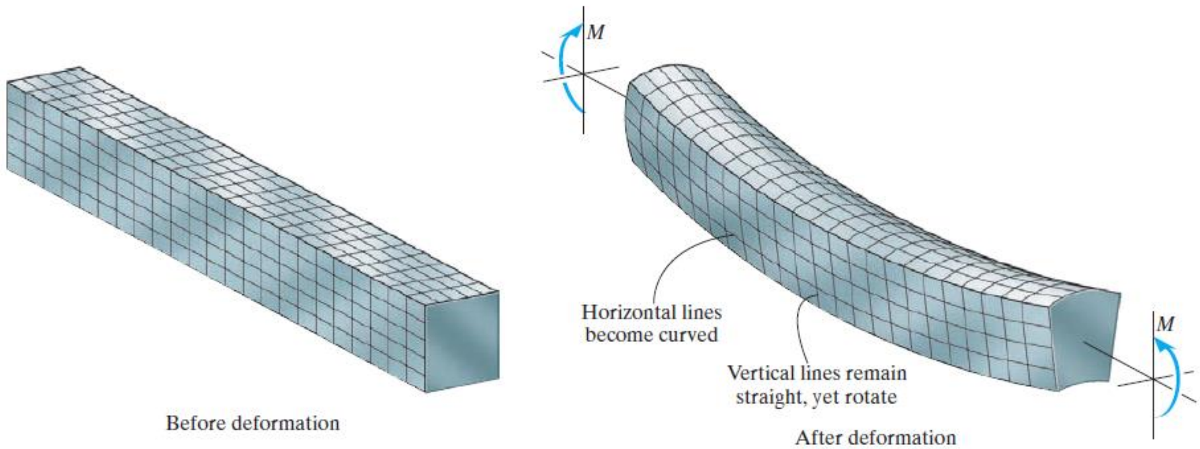


w0
w0L/2
w0L²/3
V
w0L/2
x
M
x
−w0L²/3



15 kN
5 kN/m
80 kN·m
A
B
C
5 m
5 m
5.75 kN
34.25 kN
V (kN)
5.75
x(m)
−9.25
−34.25
M (kN·m)
108.75
80
x(m)

Bending Deformation of a Straight Member

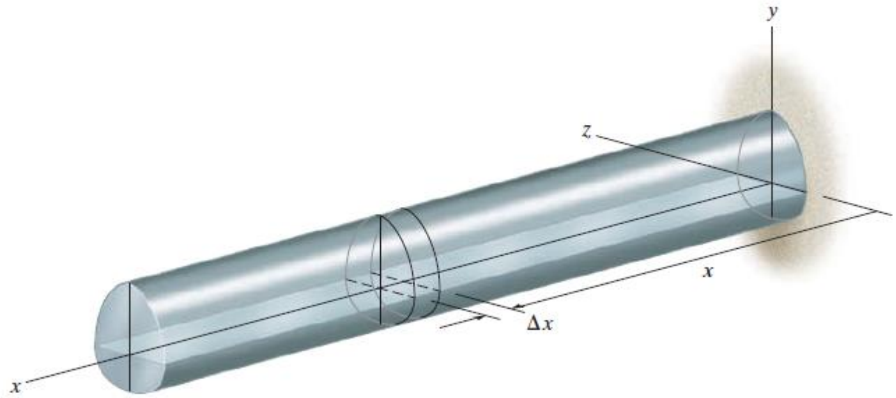


(a) (b)

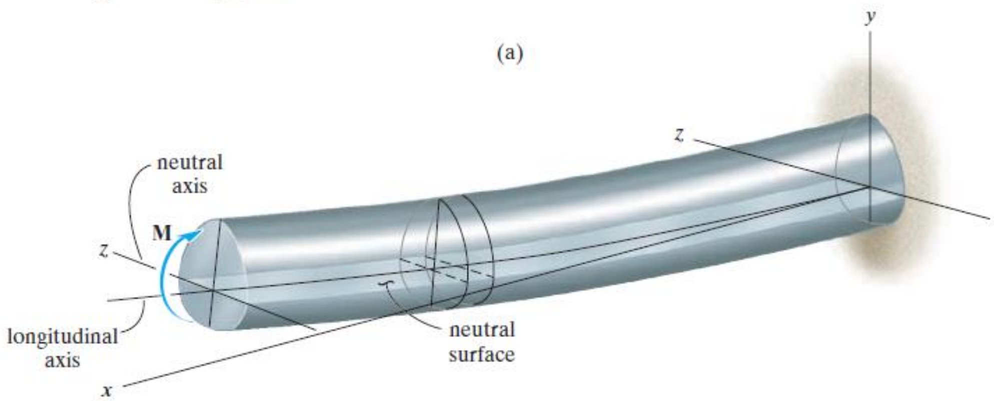
Consider, for example, the undeformed bar in the figure, which is marked with longitudinal and transverse grid lines. When a bending moment is applied, we notice that the **longitudinal lines become *curved* and the vertical transverse lines *remain straight* and yet undergo a *rotation***.

The bending moment causes the material within the ***bottom* portion of the bar to *stretch* and the material within the *top* portion to *compress***. Consequently, between these two regions there must be a surface, called the ***neutral surface***, in which longitudinal fibers of the material **will not undergo a change in length**.

From the above observations, we can draw the following conclusion, all ***cross sections*** of the beam ***remain plane*** and perpendicular to the longitudinal axis during the deformation.



(a)



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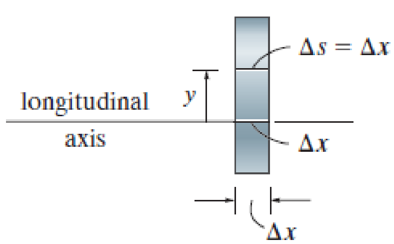
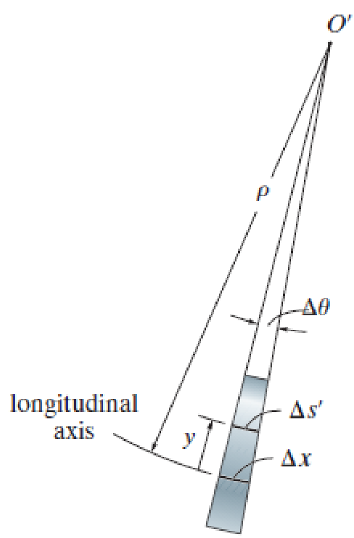


Undeformed element

(a)



Deformed element

(b)

Notice that any line segment located on the neutral surface, does not change its length, whereas any line segment located at the arbitrary distance y above the neutral surface, will contract and become after deformation. By definition, the normal strain along is determined as

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

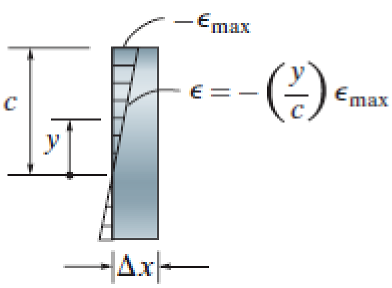
Substituting $\Delta s' = (\rho - y)\Delta\theta$, and $\Delta \mathrm{s} = \rho\Delta\theta$ in the above expression for strain,

$$\epsilon = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

Or

$$\epsilon = -\frac{y}{\rho}$$

This important result indicates that the longitudinal normal strain of any element within the beam depends on its location y on the cross section and the radius of curvature of the beam's longitudinal axis at the point. In other words, for any specific cross section, the ***longitudinal normal strain will vary linearly*** with y from the neutral axis. For points above the neutral axis (y is positive, the strain is (-ve) or contraction and those points below the neutral axis (y is +ve) the strain is (+ve) or elongation.



Normal strain distribution

Here the maximum strain occurs at the outermost fiber, located a distance of $y = c$ from the neutral axis. Using the strain equation and by division,

$$\frac{\epsilon}{\epsilon_{\max}} = -\left(\frac{y/\rho}{c/\rho}\right)$$

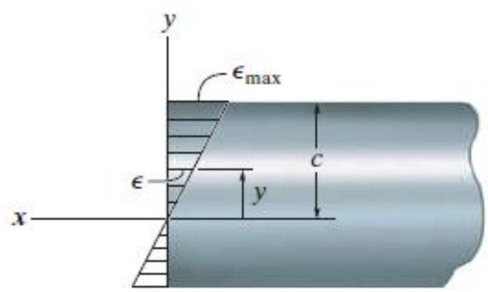
So that

$$\epsilon = -\left(\frac{y}{c}\right)\epsilon_{\max}$$

The Flexure formula

Assuming the material is linear elastic (Hooke's law) or $\sigma = E\epsilon$

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$



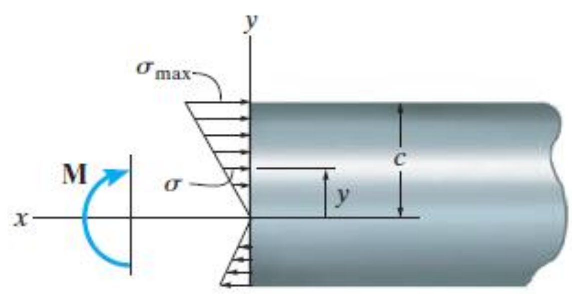
Normal strain variation
(profile view)

(a)

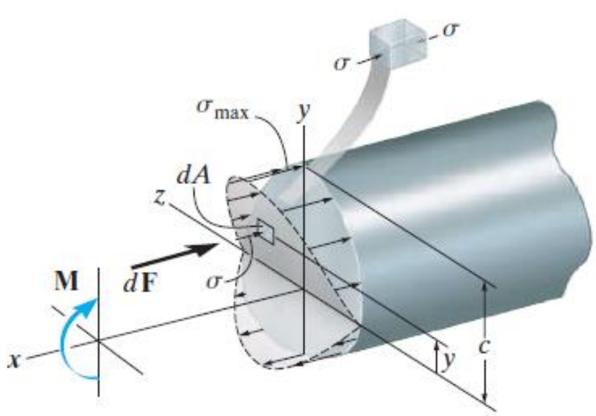
$$F_R = \Sigma F_x; \qquad 0 = \int_A dF = \int_A \sigma\, dA$$

$$= \int_A -\left(\frac{y}{c}\right)\sigma_{\max}\, dA$$

$$= \frac{-\sigma_{\max}}{c}\int_A y\, dA$$



Bending stress variation
(profile view)



Bending stress variation

Since σ_{max}/c is not equal to zero, then

$$\int_A y\, dA = 0$$

Or the first moment of the cross-section equals zero.

In other words, the first moment of the member's cross-sectional area about the neutral axis must be zero. This condition can only be satisfied if the *neutral axis* is also the horizontal *centroidal axis* for the cross section.

We can determine the stress in the beam from the requirement that the resultant internal moment ***M*** must be equal to the moment produced by the stress distribution about the neutral axis. The moment of a differential force dF is $dM = dF\, y$ and since $dF = \sigma dA$

$$(M_R)_z = \Sigma M_z; \quad M = \int_A y\, dF = \int_A y(\sigma\, dA) = \int_A y\left(\frac{y}{c}\sigma_{max}\right) dA$$

or

$$M = \frac{\sigma_{max}}{c}\int_A y^2\, dA$$

Which can be written as

$$\boxed{\sigma_{max} = \frac{Mc}{I}}$$

Where ***I*** is the moment of inertial of the cross section about the neutral axis. Substitute

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

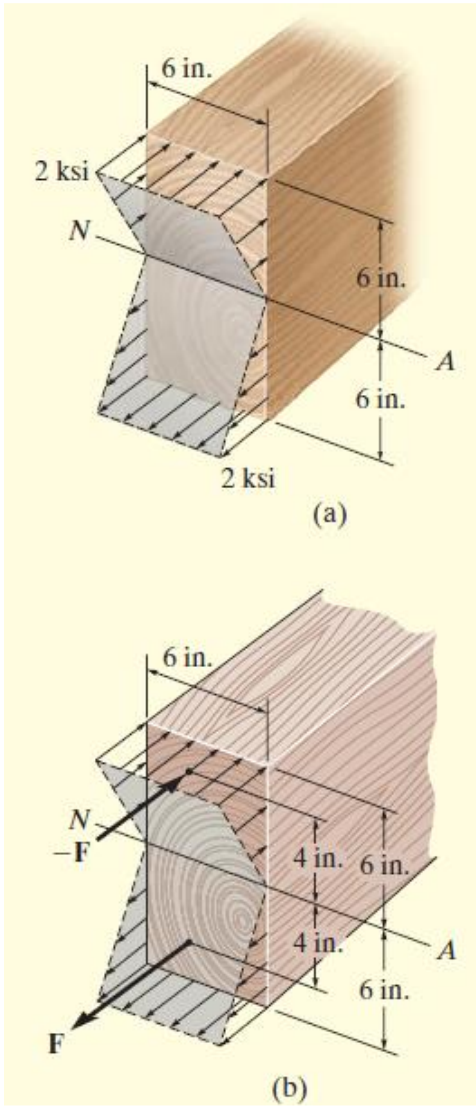
$$-\left(\frac{c}{y}\right)\sigma = \frac{Mc}{I}$$

From which,

$$\sigma = -\frac{My}{I}$$

Problems

A beam has a rectangular cross section and is subjected to the stress distribution shown in Figure. Determine the internal moment **M** at the section caused by the stress distribution (a) using the flexure formula, (b) by finding the resultant of the stress distribution using basic principles.



SOLUTION

Part (a).

$$\sigma = \frac{Mc}{I}$$

$$I = \frac{6 * 12^3}{12} = 864\ in^4$$

$$M = \frac{\sigma\, I}{c} = \frac{2 * 864}{6} = 288\ k.in$$

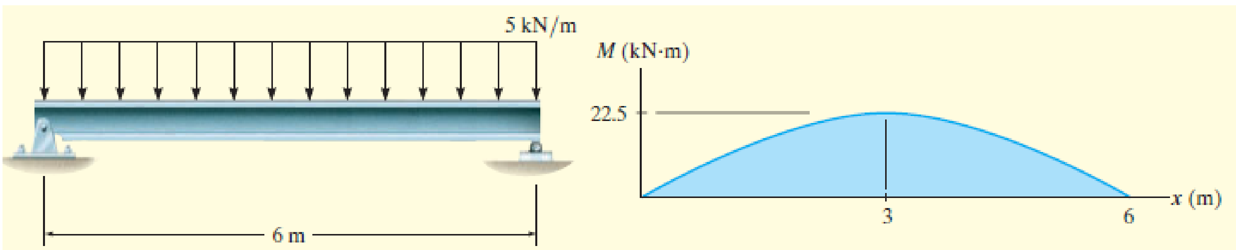
Part (b).

$$F = \frac{1}{2}(6\text{ in.})(2\text{ kip/in}^2)(6\text{ in.}) = 36\text{ kip}$$

Furthermore, they act through the *centroid* of each volume, i.e. ($\frac{2}{3} * 6in = 4in$). Then

$$M = F * 4\ in = 36\ kip * 8\ in = 288\ k.in$$

The simply supported beam in Figure has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



(a) (c)

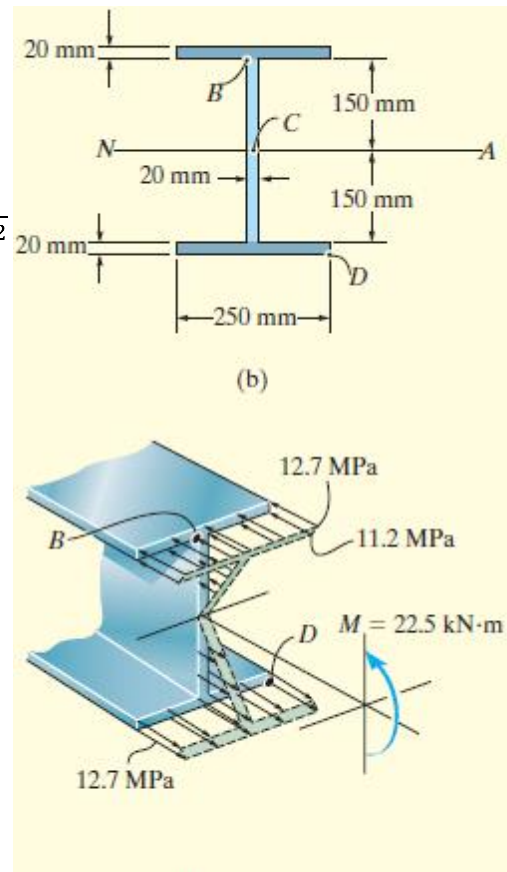
Solution:

$$I = \frac{250 * 340^3}{12} - \frac{230 * 300^3}{12} = 301.33(10)^{-6}\ mm^4$$

$$\sigma_{max} = \frac{M_{max}C}{I} = \frac{22.5(10)^3(N) * 170(mm)}{301.33^{-6}(mm^4)} = 12.7\frac{N}{mm^2}$$

$$= 12.7\ MPa$$

NOTE: It is, in many times, easier to use N and mm units and the stress result will be in MPa.



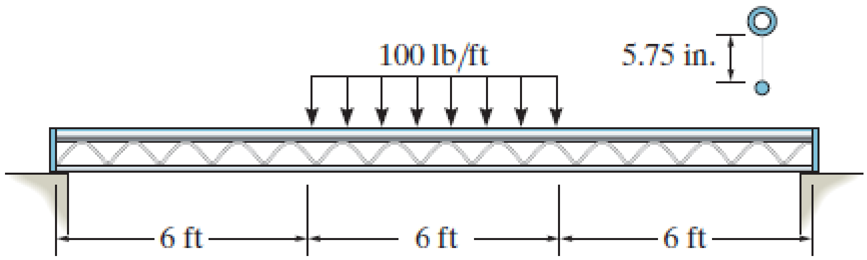
(b) (d)

At point B on the cross section, the stress is

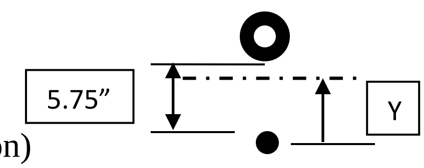
$$\sigma_B = -\frac{M_{max}y_B}{I} = -\frac{22.5(10)^3(N) * 150(mm)}{301.33^{-6}(mm^4)}$$

$$= -11.2\frac{N}{mm^2} = 11.2\ MPa\ Compression.$$

The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 1 in. and thickness of $\frac{3}{16}$ in and the bottom member is a solid rod having a diameter of $\frac{1}{2}$ in.



Solution:



1) Find max. bending moment M_{max}.

2) Locate the N.A. (which is the centroid of the whole section)

Interior radius of the pipe is $1 - 2 * \frac{3}{16} = \frac{10}{16}$"

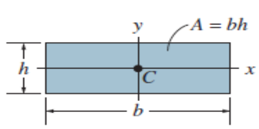
Take the first moment of the areas about the centroid of the solid rod:

$$Y = \frac{\Sigma a_i y_i}{\Sigma a_i} = 4.6091"$$

3) Find the moment of inertia about the N.A. $I_{NA} = 5.9271\ in^4$

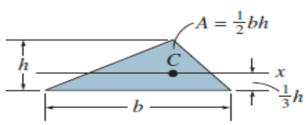
4) $\sigma_{max} = \frac{MC}{I} = 22.1\ ksi$ and C is the distance from the N.A. to the farthest point away from the N.A.

Note: You need to memorize the following formulas:



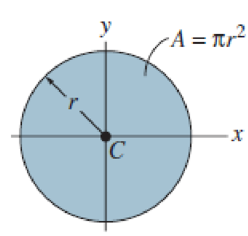
Rectangular area

$I_x = \frac{1}{12}bh^3$
$I_y = \frac{1}{12}hb^3$



Triangular area

$I_x = \frac{1}{36}bh^3$



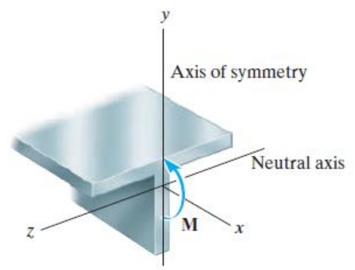
Circular area

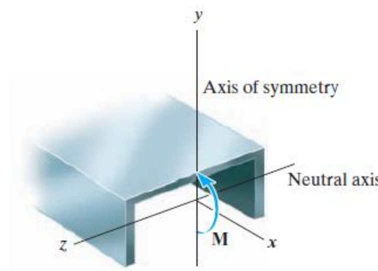
$I_x = \frac{1}{4}\pi r^4$
$I_y = \frac{1}{4}\pi r^4$

Unsymmetric Bending

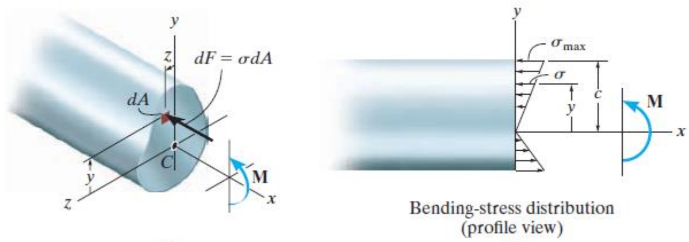
Case 1) The applied moment is in the direction of the principal axis Z and the section is not-symmetric.

The sections considered so far are symmetric sections like the T-beam and the channel shown to the right.





But consider the section show in figure shown below. This is an



(a)

Bending-stress distribution (profile view)

(b)

unsymmetric section, as result there will be bending moment about the Y-axis in addition to the bending moment about the Z-axis. The moment about the X-axis is zero. Why?

So the balance equations are :

1) Sum of the forces along X-axis is zero

2) Sum of bending moments about the Z-axis should equal applied moment about Z-axis

3) Sum of bending moments about the Y-axis should equal =0 (No applied moment about Y-axis = 0).

Mathematically, these can be translated as follows:

$$F_R = \Sigma F_x; \qquad 0 = -\int_A \sigma \, dA$$

$$(M_R)_y = \Sigma M_y; \qquad 0 = -\int_A z\sigma \, dA$$

$$(M_R)_z = \Sigma M_z; \qquad M = \int_A y\sigma \, dA$$

First equation as before

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

$$F_R = \Sigma F_x; \qquad 0 = \int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\left(\frac{y}{c}\right)\sigma_{\max} \, dA$$

$$\frac{-\sigma_{\max}}{c} \int_A y \, dA$$

Since $\sigma_{\max}/c$ is not equal to zero, then

$$\int_A y \, dA = 0$$

Again, this condition can only be satisfied if the *neutral axis* is also the horizontal *centroidal axis* for the cross section.

For the second equation, substitute the following equation for stress and integrate

$$\sigma = -\left(\frac{y}{c}\right) \sigma_{max}$$

$$0 = \frac{-\sigma_{\max}}{c} \int_A yz \, dA$$

which requires

$$\int_A yz \, dA = 0$$

This term is called ***product of inertia*** and this condition is satisfied if either the Z or Y axes are axes of symmetry or these axes are principal axes. Here these axes are NOT axes of symmetry, then the Z and Y axes must be principal axes.

For the third equation, substitute the following equation for stress and integrate

$$\sigma = -\left(\frac{y}{c}\right) \sigma_{max}$$

Which will lead to the same result before

$$\sigma = -\frac{M_z y}{I_z}$$

What is different from before? The difference is that Mz, y and Iz should be calculated about the principal axis Z.

Case 2) The section is symmetric, but the loading (moment) is applied about two principal axes Z and Y.

The stress can be determined by combining the stresses from bending around both axes as follows:

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Case 3) The section is NOT symmetric, and the loading is about any two perpendicular axes NOT necessarily principal axes.

Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$, as shown, when the x, y, z axes pass through the centroid of the cross section. If the material is linear-elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma dA, M_y = \int_A z\sigma dA, M_z = \int_A - y\sigma dA$, one can determine the constants a, b, and c, and show that the normal stress can be determined from the equation

$$\sigma = \left[-\left(M_z I_y + M_y I_{yz}\right)y + \left(M_y I_z + M_z I_{yz}\right)z\right]/\left(I_y I_z - I_{yz}^2\right)$$

where the moments and products of inertia are defined in Appendix A of the book "Mechanics of Material for Hibbeler".

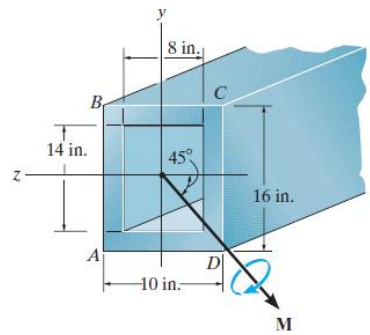
If the section is not symmetric and $M_y = 0$ but we do the calculations about two principal axes, the $I_{yz} = 0$, then this is case 1) and $\sigma = -\frac{M_z y}{I_z}$.

If the section is symmetric and the loading is about two axes ($M_z and\ M_y$), and we do calculations about two principal axes $I_{yz} = 0$, then this is case 2 and $\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

Problems:

The beam is subjected to a bending moment of M=20 Kips.ft directed as shown. Determine the maximum bending stress in the beam.

Solution:



The y and z components of M are negative, Fig. a. Thus,

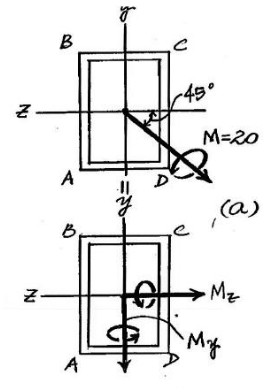
$$M_y = -20 \sin 45° = -14.14 \text{ kip} \cdot \text{ft}$$

$$M_z = -20 \cos 45° = -14.14 \text{ kip} \cdot \text{ft}.$$

The moments of inertia of the cross-section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(16)(10^3) - \frac{1}{12}(14)(8^3) = 736 \text{ in}^4$$

$$I_z = \frac{1}{12}(10)(16^3) - \frac{1}{12}(8)(14^3) = 1584 \text{ in}^4$$



By inspection, the bending stress occurs at corners A and C are

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

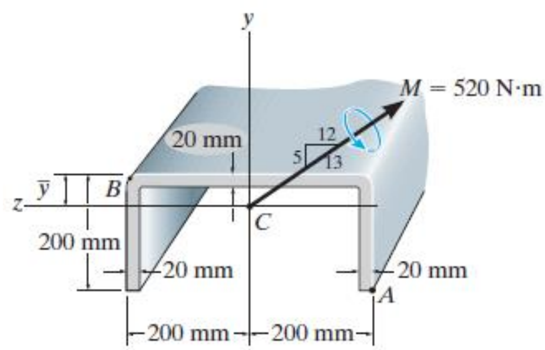
$$\sigma_{\max} = \sigma_C = -\frac{-14.14(12)(8)}{1584} + \frac{-14.14(12)(-5)}{736}$$

$$= 2.01 \text{ ksi} \quad \text{(T)}$$ **Ans.**

$$\sigma_{\max} = \sigma_A = -\frac{-14.14(12)(-8)}{1584} + \frac{-14.14(12)(5)}{736}$$

$$= -2.01 \text{ ksi} = 2.01 \text{ ksi (C)}$$ **Ans.**

If the resultant internal moment acting on the cross section of the aluminum strut has a magnitude of M=520 N.m and is directed as shown, determine the bending stress at points A and B. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined.

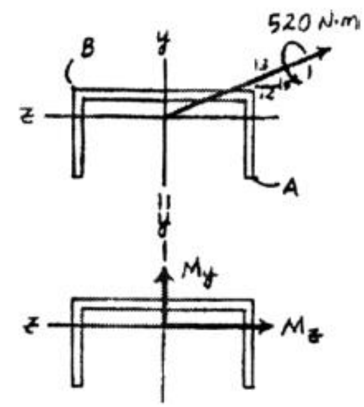


Solution:

Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$

$$Y = \frac{\Sigma a_i y_i}{\Sigma a_i} = 57.4mm\ from\ top\ edge\ of\ the\ section$$



Then calculate $I_y and\ I_z$:

$$I_y = 366.627 * 10^6 mm^4$$

$$I_z = 57.6014 * 10^6 mm^4$$

Maximum Bending Stress: Applying the flexure formula for biaxial at points *A* and *B*

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa}\ (\textbf{C}) \qquad \textbf{Ans.}$$

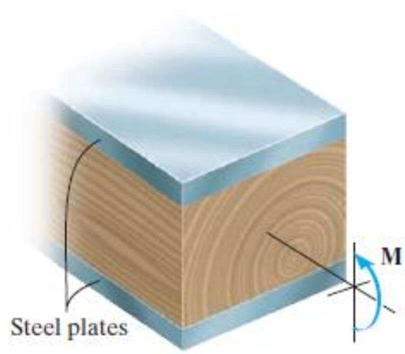
$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa}\ (\textbf{T}) \qquad \textbf{Ans.}$$

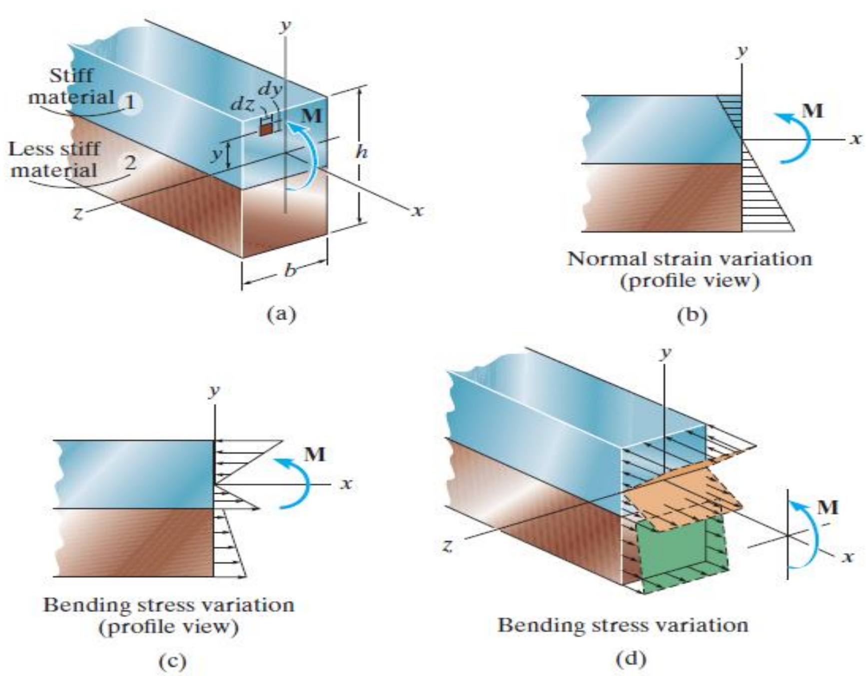
Composite Beams: (Skip for now and time permits, will be given later)

Beams constructed of two or more different materials are referred to as composite beams. For example, a beam can be made of wood with straps of steel at its top and bottom, Figure. Engineers purposely design beams in this manner in order to develop a more efficient means for supporting loads.

Since the flexure formula was developed only for beams having homogeneous material, this formula cannot be applied directly to determine the normal stress in a composite beam. In this section, however, we will develop a method for modifying or "transforming" a composite beam's cross section into one made of a single material. Once this has been done, the flexure formula can then be used for the stress analysis.



This solution method is called "Transformed section method" which transforms the beam into one made of a *single material. The key point in this this method is that the plane sections will remain plane and as a result the strain will vary linearly over the cross section.*



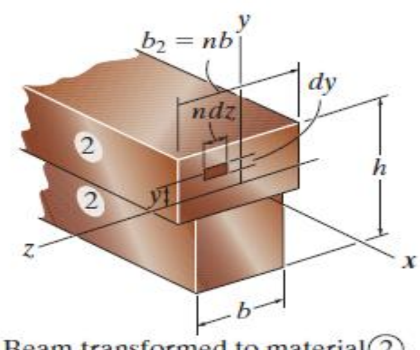
Consider a beam made of two different materials with two different Moduli of elasticity, the upper one is stiffer than the lower one (E1>E2). The strain varies linearly over the cross section as in the figure below(b). However, the stress will vary linearly over the upper material but since there is a sudden change in the material at the interface between the two materials, there will be a sudden change in the stress distribution as shown in figures (c) and (d).

First assume that we want to use material 2 for the whole section and replace material 1 in Fig(a). We don't know if the new area will be wider or narrower until we calculate it such that the carried force should be the same by the new area as by the original one.

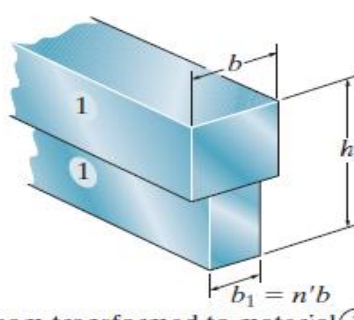
Let's assume the new width is ($n\ dz$) then since the carried force must be the same:

$$E_1 \epsilon dzdy = E_2 \epsilon (ndz)dy$$
$$n = \frac{E_1}{E_2}$$

This dimensionless number n is called the ***transformation factor***. It indicates that the cross section, having a width b on the original beam, Fig. a, must be **increased** in width to $b_2 = nb$ (since E1>E2) in the region where material 1 is being transformed into material 2, Fig. e.



Beam transformed to material ②
(e)



Beam transformed to material ①
(f)

In a similar manner, if the less stiff material 2 is transformed into the stiffer material 1, the cross section will look like that shown in Fig. f. Here the width of material 2 has been changed to $b1 = \acute{n}b$ where $\acute{n} = \frac{E2}{E1}$ .In this case the transformation factor will be less than one since E1>E2. In other words, we need less of the stiffer material to support the moment.

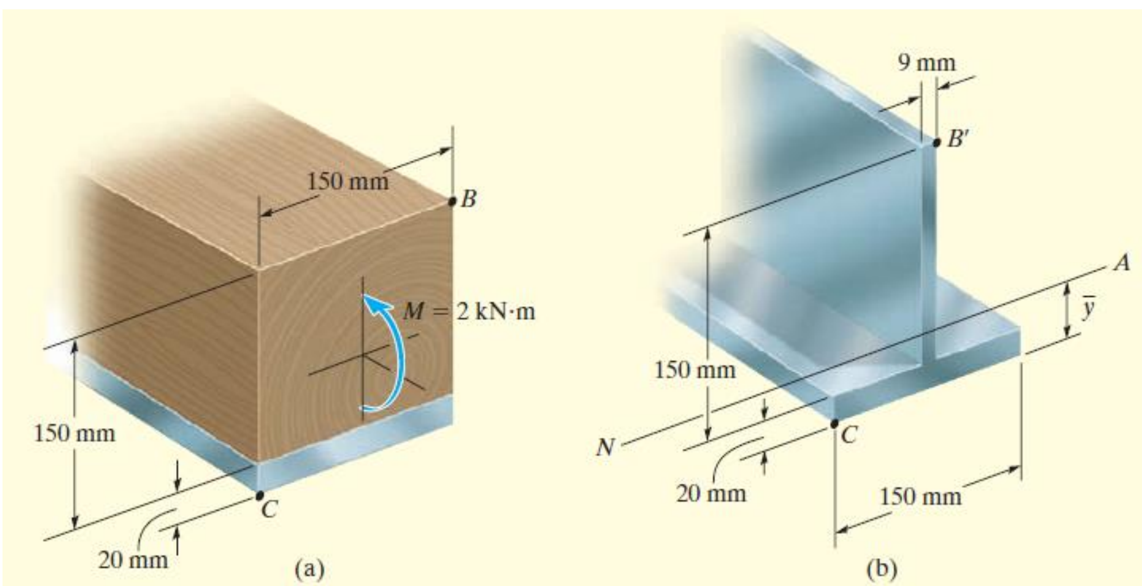
The stress on the actual stress σ on the material before transformation is calculated from the previous equation or the force on the transformed section is the same as the force on the original material.

$$dF = \sigma dA = \sigma' dA'$$
$$\sigma dzdy = \sigma' ndzdy$$
$$\sigma = n\sigma'$$

Where σ' is the stress on the transformed area and σ is the stress on the original area.

Problem:

A composite beam is made of wood and reinforced with a steel strap located on its bottom side. It has the cross-sectional area shown in Fig. a. If the beam is subjected to a bending moment of $M = 2\ kN.m$ determine the normal stress at points B and C. Take E_w = 12 GPa and E_{st} = 200 GPa.



Solution:

Although the choice is arbitrary, here we will transform the section into one made entirely of steel.

The width of transformed wood into steel is:

$$b_{st} = n\, b_w = \frac{12GPa}{200GPa}(150\, mm)$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{[0.01\text{m}](0.02\text{m})(0.150\text{m}) + [0.095\text{m}](0.009\text{m})(0.150\text{m})}{0.02\text{m}(0.150\text{m}) + 0.009\text{m}(0.150\text{m})} = 0.03638\text{m}$$

$$I_{NA} = 9 * \frac{(150-\bar{y})^3}{3} + 150 * \frac{\bar{y}^3}{3} - (150-9) * \frac{(\bar{y}-20)^3}{3} I_{NA}$$
$$= 9 * \frac{(150-\bar{y})^3}{3} + 150 * \frac{\bar{y}^3}{3} - (150-9) * \frac{(\bar{y}-20)^3}{3}$$
$$= 9.358 * 10^6 mm^4$$

$$\sigma_{B'} = \frac{2(10^3)\text{N}\cdot\text{m}(0.170\text{m} - 0.03638\text{m})}{9.358(10^{-6})\text{m}^4} = 28.6\text{MPa}$$

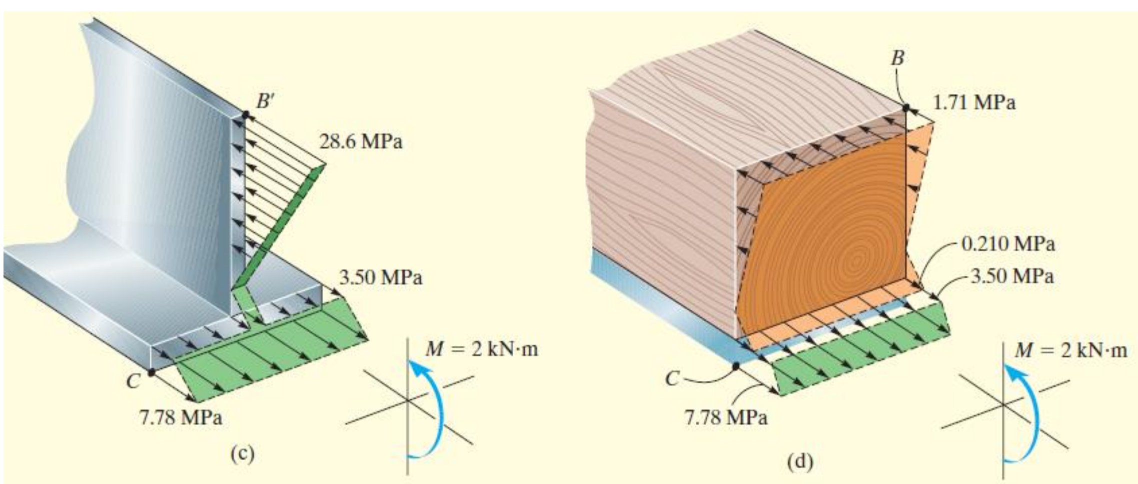
$$\sigma_C = \frac{2(10^3)\text{N}\cdot\text{m}(0.03638\text{m})}{9.358(10^{-6})\text{m}^4} = 7.78\text{MPa}$$

The normal-stress distribution on the transformed (all steel) section is shown in Fig. *c*.

The normal stress in the wood at *B* in Fig. a, is determined as,

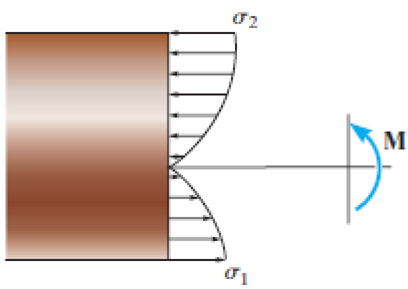
$$\sigma_B = n\sigma_{B'} = \frac{12\text{GPa}}{200\text{GPa}}(28.56\text{MPa}) = 1.71\text{MPa}$$

Using these concepts, show that the normal stress in the steel and the wood at the point where they are in contact is $\sigma_{st} = 3.5\, MPa$ and $\sigma_w = 0.21 MPa$ respectively. The normal-stress distribution in the actual beam is shown in Fig. *d*.



Inelastic bending: (Skip for now and time permits, will be given later)

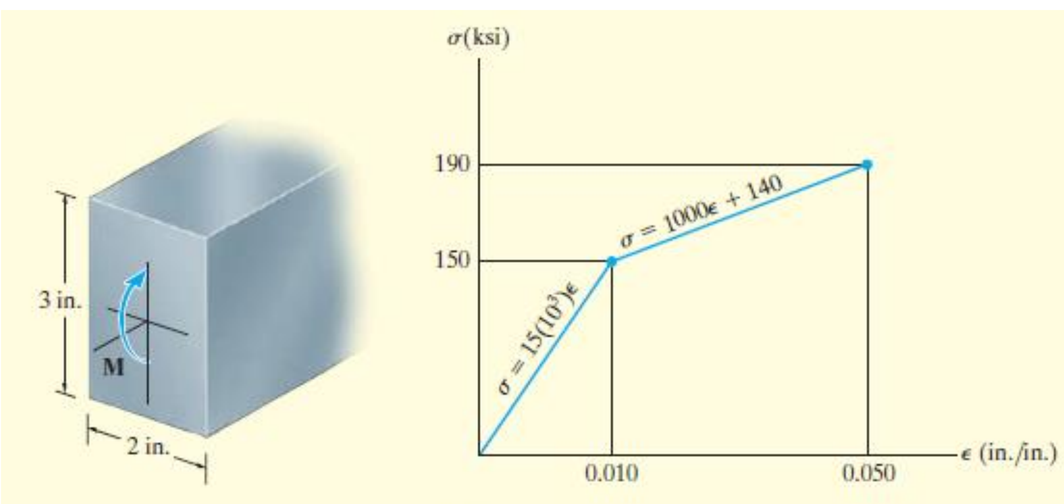
This trial-and-error procedure is obviously very tedious, and fortunately it does not occur very often in engineering practice. Most beams are symmetric about two axes, and they are constructed from materials that are assumed to have similar tension-and-compression stress–strain diagrams. Whenever this occurs, the neutral axis will pass through the centroid of the cross section, and the process of relating the stress distribution to the resultant moment is thereby simplified.



Stress distribution
(profile view)

Problem:

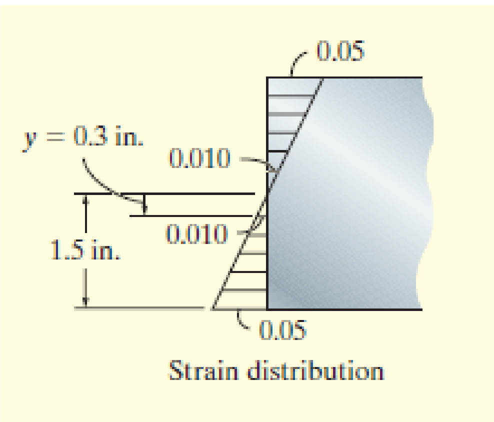
The beam in Fig. *a* below is made of an alloy of titanium that has a stress–strain diagram that can in part be approximated by two straight lines. If the material behavior is the *same* in both tension and compression, determine the bending moment that can be applied to the beam that will cause the material at the top and bottom of the beam to be subjected to a strain of 0.050 in./in.



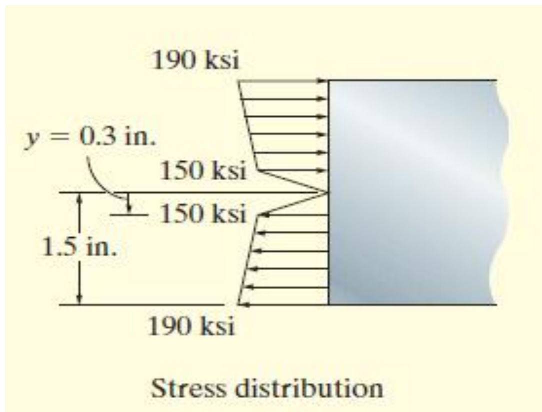
(a)

Solution I:

By inspection of the stress–strain diagram, the material is said to exhibit "**elastic-plastic** behavior with **strain hardening**." Since the cross section is symmetric and the tension–compression diagrams are the same, the neutral axis must pass through the centroid of the cross section. The strain distribution, which is always linear, is shown in Fig b. Below, In particular, the point where maximum elastic strain (0.01 in/in) occurs has been determined by proportion, such that 0.05/1.5 in=0.01/y then y=0.3 in. The stress distribution is shown in Fig (c) below.

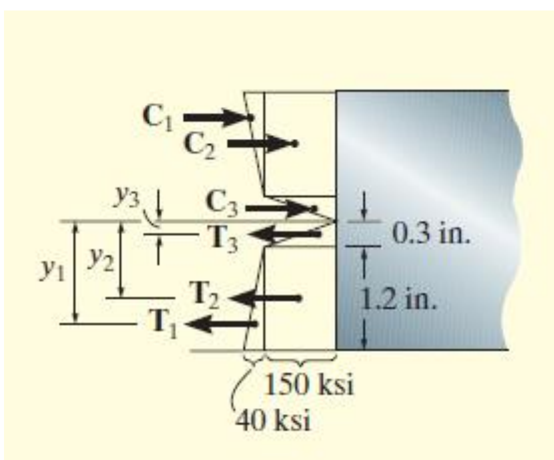


(b)



(c)

The resultant forces of the parts of the stress diagram are shown in Fig (d) below.



(d)

$$T_1 = C_1 = \frac{1}{2}(1.2\text{ in.})(40\text{ kip/in}^2)(2\text{ in.}) = 48\text{ kip}$$

$$y_1 = 0.3\text{ in.} + \frac{2}{3}(1.2\text{ in.}) = 1.10\text{ in.}$$

$$T_2 = C_2 = (1.2\text{ in.})(150\text{ kip/in}^2)(2\text{ in.}) = 360\text{ kip}$$

$$y_2 = 0.3\text{ in.} + \frac{1}{2}(1.2\text{ in.}) = 0.90\text{ in.}$$

$$T_3 = C_3 = \frac{1}{2}(0.3\text{ in.})(150\text{ kip/in}^2)(2\text{ in.}) = 45\text{ kip}$$

$$y_3 = \frac{2}{3}(0.3\text{ in.}) = 0.2\text{ in.}$$

The moment of the section will be calculated to be:

$$M = 2[48\text{ kip }(1.10\text{ in.}) + 360\text{ kip }(0.90\text{ in.}) + 45\text{ kip }(0.2\text{ in.})]$$
$$= 772\text{ kip}\cdot\text{in.} \qquad \textit{Ans.}$$

SOLUTION II:

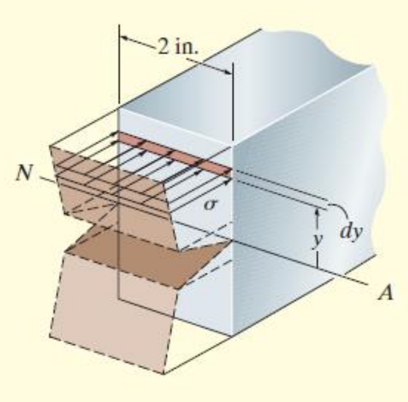
Rather than solving the problem semigraphically, it is also possible to find the moment analytically.

$$\sigma = 15(10^3)\epsilon = 15(10^3) * \frac{0.01}{0.3}y = 500y \quad for\ 0 \le y \le 0.3$$

$$\sigma = 1000\epsilon + 140 = 1000 * \frac{0.05}{1.5}y + 140 = 33.3y + 140 \quad for\ 0.3 \le y \le 1.5$$

$$dM = y(\sigma\, dA) = y\sigma(2\,dy)$$

$$M = 2\left[2\int_0^{0.3\text{ in.}} 500y^2\,dy + 2\int_{0.3\text{ in.}}^{1.5\text{ in.}} (33.3y^2 + 140y)\,dy\right]$$
$$= 772\text{ kip}\cdot\text{in.} \qquad \textit{Ans.}$$



(e)

Conceptual problems:

Problem 1:

Hurricane winds caused failure of this highway sign by bending the supporting pipes at their connections with the column.

1) Search for a formula to calculate the wind pressure from the Internet (wind pressure P (Pa) is a function of the basic wind speed V(m/sec).

2) Search for the basic wind speed in your area zone.

3) Assume reasonable dimensions of the pipes supporting the sign and the sign (For example outer diameter 2 in =5 cm or 2.5 in =6.25 cm or 3 in 7.5 cm). The thickness could be ¼ in=0.625 cm or ½ in=1.25 cm.

Note: when designing a steel section make sure it's available in the market or you need to build it using welding for example.

4) Estimate the length of the supporting pipes (For example 1.5m or 2 m)

5) Calculate maximum moment of the wind pressure on the pipes) and ignore the weight of the pipes and the sign.

6) Check maximum stress on the pipes using the flexure formula and check against maximum yield stress of the steel type A36. You need to search and find this allowable stress for this type of steel.

Solution:

1) Wind pressure $P = 0.613\,V^2$

2) In Iraq, basic wind speed is 44 m/sec

3) Assume the supporting pipes are of 2 in=5 cm outer diameter and ¼ in=0.625 cm thick. And the sign dimensions are 0.750mx1m.

4) Assume length of the supporting pipes is 2m.

5) Wind pressure is $P = 0.613 * 44^2 = 1{,}186.768\,\frac{N}{m^2}$ and maximum bending moment is

$1{,}186.768 * 0.75 * 1 * 1.5 = 1{,}335.114\ N.m$. This moment is carried by two pipes so each one carries 1335.114/2=667.557 N.m.

6 $I_{NA} = \frac{0.05^4\pi}{64} - \frac{(0.05-0.00625)^4\pi}{64} = 0.127(10^{-6})m^4$ then

$$\sigma = \frac{MC}{I_{NA}} = 667.557 * \frac{\frac{0.075}{2}}{0.127(10^{-6})} = 131.4\ MPQ < \sigma_{yield} = 250\ Mpa\ O.K.$$

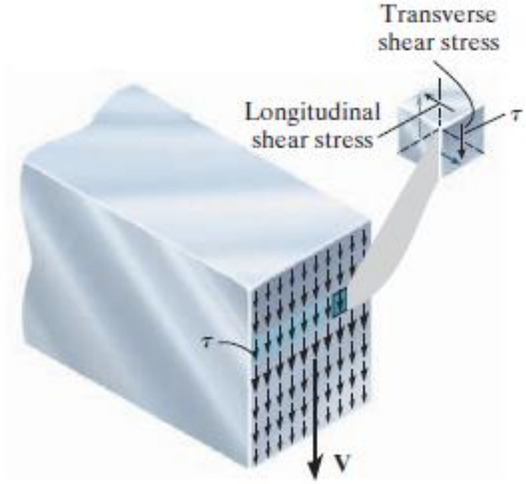
Problem 2:

Draw bending moment diagram for the ship boom in the figure below and explain why it tapers this way.

Solution: DIY

Transverse Shear:

In general, a beam will support both shear and moment. The shear **V** is the result of a transverse shear-stress distribution that acts over the beam's cross section. Due to the complementary property of shear, however, this stress will create corresponding longitudinal shear stresses which will act along longitudinal planes of the beam as shown in Fig. below.



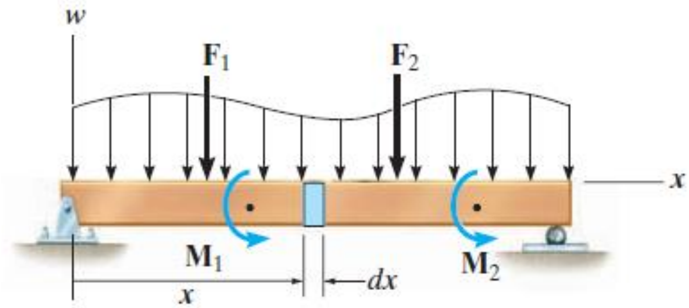
To illustrate this effect, consider the beam to be made from three boards, Fig. a. If the top and bottom surfaces of each board are smooth, and the boards are *not* bonded together, then application of the load **P** will cause the boards to *slide* relative to one another when the beam deflects. However, if the boards are bonded together, then the longitudinal shear stresses acting between the boards will prevent their relative sliding, and consequently the beam will act as a single unit, Fig. b.

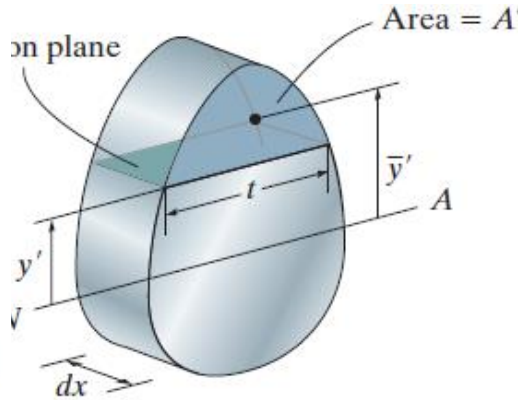
Boards not bonded together
(a)

Boards bonded together
(b)

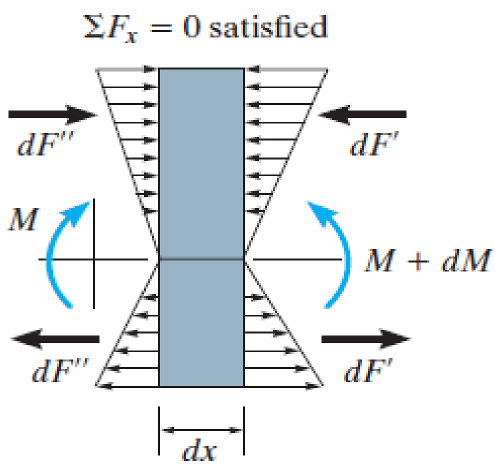
The shear formula

Because the strain distribution for shear is not easily defined, as in the case of axial load, torsion, and bending, we will develop the shear formula in an indirect manner. To do this we will consider the ***horizontal force equilibrium*** of a portion of the element taken from the beam in Fig.a.



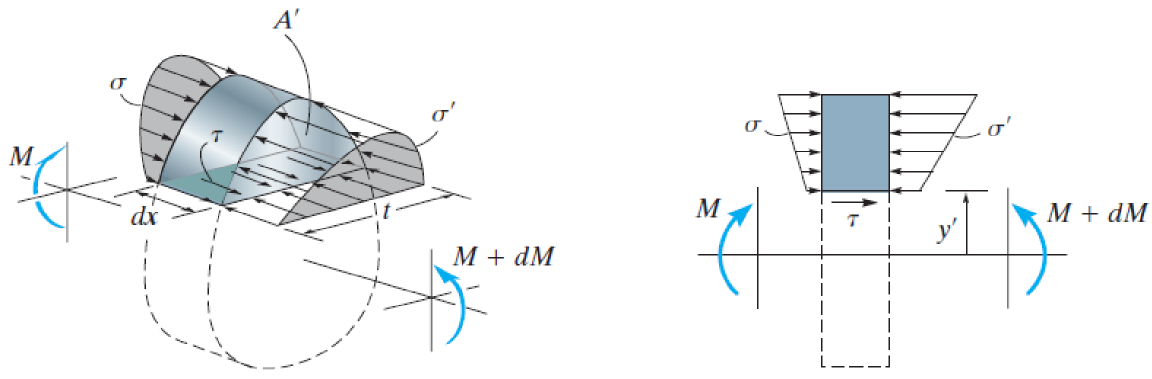


(a)



(b)

The equilibrium is satisfied for the element in Fig b above. However, consider the shaded top *portion* of the element that has been sectioned at $\acute{y}$ from the neutral axis Fig (c) below.



Three-dimensional view (c) Profile view

$\Sigma F_x = 0$ will NOT be satisfied unless a shear stress acts on the bottom face of the element, then we have,

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad \int_{A'} \sigma' \, dA' - \int_{A'} \sigma \, dA' - \tau(t\,dx) = 0$$

$$\int_{A'} \left(\frac{M + dM}{I}\right) y \, dA' - \int_{A'} \left(\frac{M}{I}\right) y \, dA' - \tau(t\,dx) = 0$$

$$\left(\frac{dM}{I}\right) \int_{A'} y \, dA' = \tau(t\,dx)$$

Solving for τ, we get

$$\tau = \frac{1}{It}\left(\frac{dM}{dx}\right) \int_{A'} y \, dA'$$

Since $V = \frac{dM}{dx}$ (shear force equals rate of change of bending moment or slope of bending moment diagram)

Call $Q = \int_{\acute{A}} y \, d\acute{A}$ that represents the first moment about N.A. for an area below which shear stress is determined.

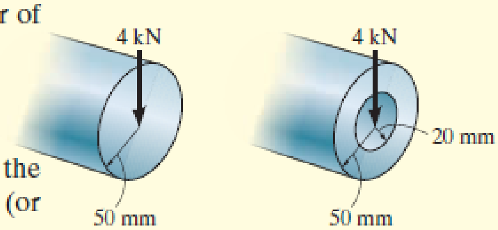
t: is the thickness of the beam. Then the shear formula becomes.

$$\tau = \frac{VQ}{It}$$

This stress is assumed to be constant and therefore *averaged* across the width t of the member.

Examples:

The solid shaft and tube shown in Fig. 7–9*a* are subjected to the shear force of 4 kN. Determine the shear stress acting over the diameter of each cross section.



(a)

SOLUTION

Section Properties. Using the table on the inside front cover, the moment of inertia of each section, calculated about its diameter (or neutral axis), is

$$I_{solid} = \frac{1}{4}\pi c^4 = \frac{1}{4}\pi(0.05\text{ m})^4 = 4.909(10^{-6})\text{ m}^4$$

$$I_{tube} = \frac{1}{4}\pi(c_o^4 - c_i^4) = \frac{1}{4}\pi[(0.05\text{ m})^4 - (0.02\text{ m})^4] = 4.783(10^{-6})\text{ m}^4$$

The semicircular area shown shaded in Fig. 7–9*b*, above (or below) each diameter, represents Q, because this area is "held onto the member" by the longitudinal shear stress along the diameter.

$$Q_{solid} = \bar{y}'A' = \frac{4c}{3\pi}\left(\frac{\pi c^2}{2}\right) = \frac{4(0.05\text{ m})}{3\pi}\left(\frac{\pi(0.05\text{ m})^2}{2}\right) = 83.33(10^{-6})\text{ m}^3$$

$$Q_{tube} = \Sigma\bar{y}'A' = \frac{4c_o}{3\pi}\left(\frac{\pi c_o^2}{2}\right) - \frac{4c_i}{3\pi}\left(\frac{\pi c_i^2}{2}\right)$$

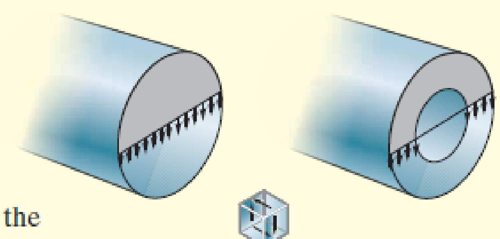
$$= \frac{4(0.05\text{ m})}{3\pi}\left(\frac{\pi(0.05\text{ m})^2}{2}\right) - \frac{4(0.02\text{ m})}{3\pi}\left(\frac{\pi(0.02\text{ m})^2}{2}\right)$$

$$= 78.0(10^{-6})\text{ m}^3$$

Shear Stress. Applying the shear formula where $t = 0.1$ m for the solid section, and $t = 2(0.03\text{ m}) = 0.06$ m for the tube, we have

$$\tau_{solid} = \frac{VQ}{It} = \frac{4(10^3)\text{ N}(83.33(10^{-6})\text{ m}^3)}{4.909(10^{-6})\text{ m}^4(0.1\text{ m})} = 679\text{ kPa} \qquad \textit{Ans.}$$

$$\tau_{tube} = \frac{VQ}{It} = \frac{4(10^3)\text{ N}(78.0(10^{-6})\text{ m}^3)}{4.783(10^{-6})\text{ m}^4(0.06\text{ m})} = 1.09\text{ MPa} \qquad \textit{Ans.}$$

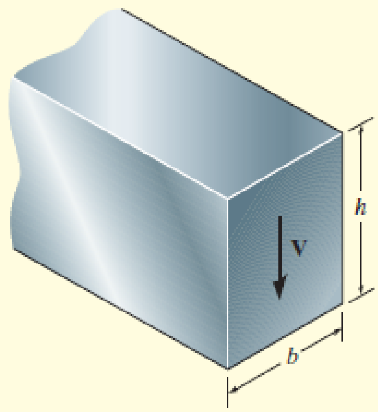


(b)

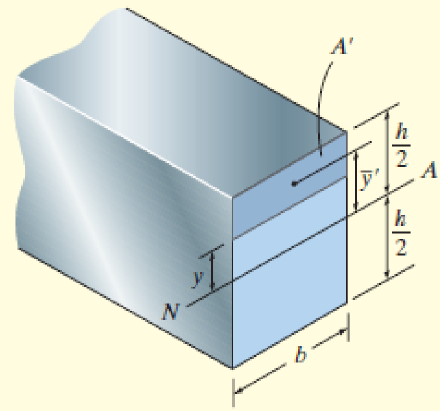
Fig. 7–9

NOTE: As discussed in the limitations for the shear formula, the calculations performed here are valid since the shear stress along the diameter is vertical and therefore tangent to the boundary of the cross section. An element of material on the diameter is subjected to "pure shear" as shown in Fig. 7–9*b*.

Determine the distribution of the shear stress over the cross section of the beam shown in Fig. 7–10*a*.



(a)



(b)

SOLUTION

The distribution can be determined by finding the shear stress at an *arbitrary height y* from the neutral axis, Fig. 7–10*b*, and then plotting this function. Here, the dark colored area A' will be used for Q.* Hence

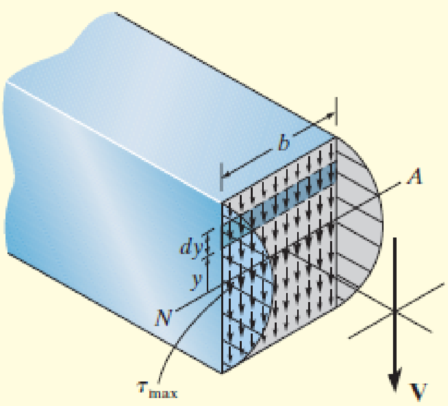
$$Q = \bar{y}'A' = \left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right]\left(\frac{h}{2} - y\right)b = \frac{1}{2}\left(\frac{h^2}{4} - y^2\right)b$$

Applying the shear formula, we have

$$\tau = \frac{VQ}{It} = \frac{V\left(\frac{1}{2}\right)[(h^2/4) - y^2]b}{\left(\frac{1}{12}bh^3\right)b} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right) \qquad (1)$$

This result indicates that the shear-stress distribution over the cross section is ***parabolic***. As shown in Fig. 7–10*c*, the intensity varies from zero at the top and bottom, $y = \pm h/2$, to a maximum value at the neutral axis, $y = 0$. Specifically, since the area of the cross section is $A = bh$, then at $y = 0$ we have

$$\tau_{\max} = 1.5\frac{V}{A} \qquad (2)$$

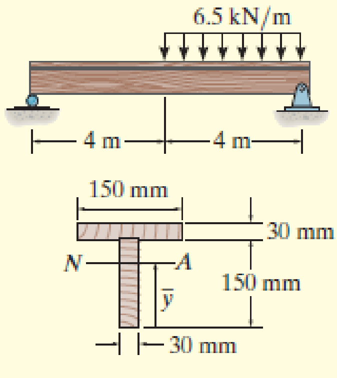


Shear–stress distribution

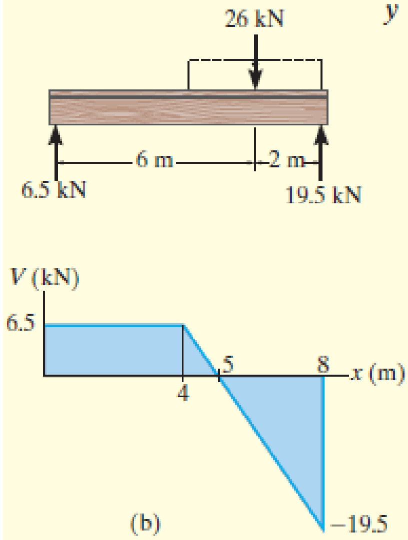
(c)

Fig. 7–10

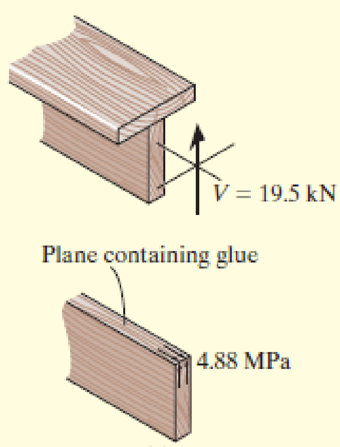
*The area below y can also be used $[A' = b(h/2 + y)]$, but doing so involves a bit more algebraic manipulation.



(a)



(b)



(c)

Fig. 7–12

The beam shown in Fig. 7–12*a* is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the seam where they are joined.

SOLUTION

Internal Shear. The support reactions and the shear diagram for the beam are shown in Fig. 7–12*b*. It is seen that the maximum shear in the beam is 19.5 kN.

Section Properties. The centroid and therefore the neutral axis will be determined from the reference axis placed at the bottom of the cross-sectional area, Fig. 7–12*a*. Working in units of meters, we have

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{[0.075\text{ m}](0.150\text{ m})(0.030\text{ m}) + [0.165\text{ m}](0.030\text{ m})(0.150\text{ m})}{(0.150\text{ m})(0.030\text{ m}) + (0.030\text{ m})(0.150\text{ m})} = 0.120\text{ m}$$

The moment of inertia, about the neutral axis, Fig. 7–12*a*, is therefore

$$I = \left[\frac{1}{12}(0.030\text{ m})(0.150\text{ m})^3 + (0.150\text{ m})(0.030\text{ m})(0.120\text{ m} - 0.075\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.150\text{ m})(0.030\text{ m})^3 + (0.030\text{ m})(0.150\text{ m})(0.165\text{ m} - 0.120\text{ m})^2\right]$$

$$= 27.0(10^{-6})\text{ m}^4$$

The top board (flange) is being held onto the bottom board (web) by the glue, which is applied over the thickness $t = 0.03$ m. Consequently A' is defined as the area of the top board, Fig. 7–12*a*. We have

$$Q = \bar{y}'A' = [0.180\text{ m} - 0.015\text{ m} - 0.120\text{ m}](0.03\text{ m})(0.150\text{ m})$$

$$= 0.2025(10^{-3})\text{ m}^3$$

Shear Stress. Using the above data and applying the shear formula yields

$$\tau_{max} = \frac{VQ}{It} = \frac{19.5(10^3)\text{ N}(0.2025(10^{-3})\text{ m}^3)}{27.0(10^{-6})\text{ m}^4(0.030\text{ m})} = 4.88\text{ MPa} \qquad \textit{Ans.}$$

The shear stress acting at the top of the bottom board is shown in Fig. 7–12*c*.

NOTE: It is the glue's resistance to this longitudinal *shear* stress that holds the boards from slipping at the right-hand support.

Combined Loading

The normal stress is

$$\sigma = \frac{P}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in.})} = 3.75 \text{ psi}$$

And the bending stress is

$$\sigma_{max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}(5 \text{ in.})}{\frac{1}{12}(4 \text{ in.})(10 \text{ in.})^3} = 11.25 \text{ psi}$$

Elements at B and C are subjected to the following combined stresses

$\sigma_B = -3.754 + 11.25 = 7.5\ MPa$ Tension

$\sigma_C == -3.754 - 11.25 = 15\ MPa$ Compression

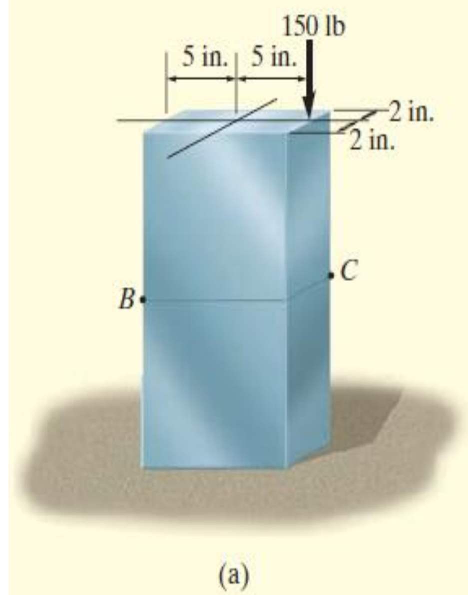
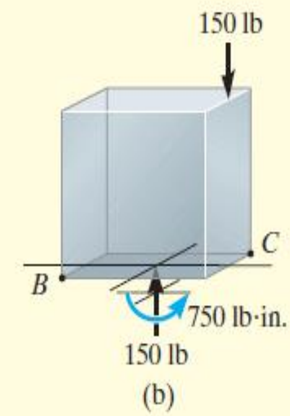
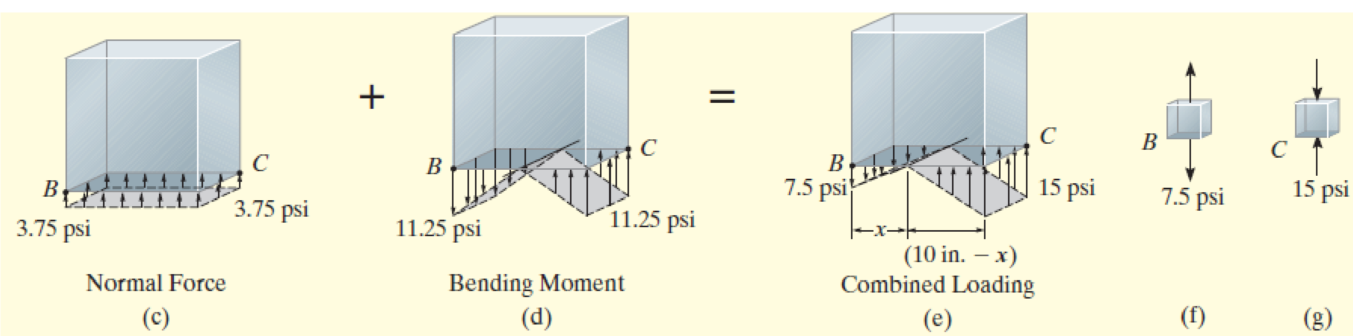


Fig. 8–3



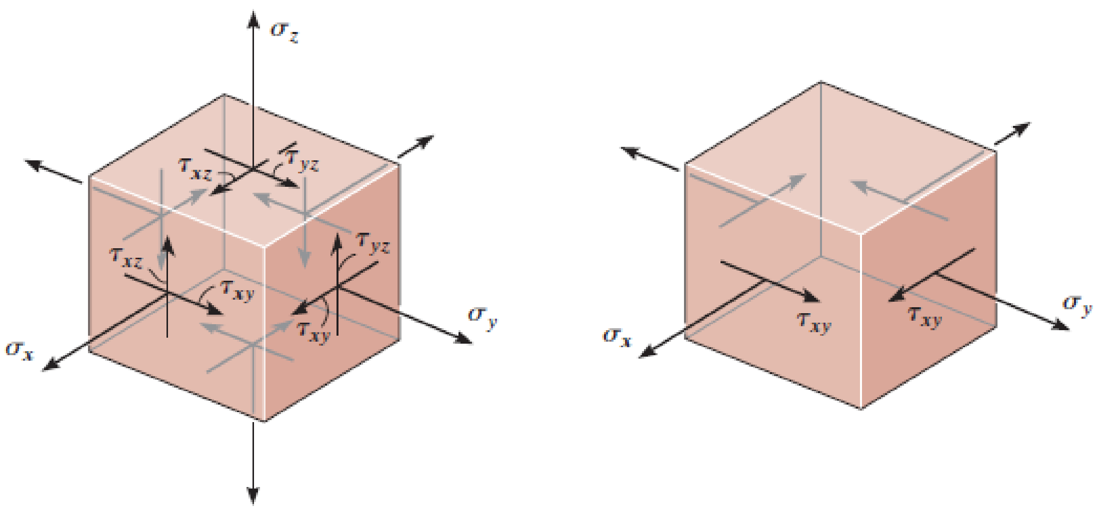


Stress Transformation

Plane stress

It was shown in Sec. 1.3 that the general state of stress at a point is characterized by *six* independent normal and shear stress components, which act on the faces of an element of material located at the point, Fig. *a*. This state of stress, however, is not often encountered in engineering practice. Instead, engineers frequently make approximations

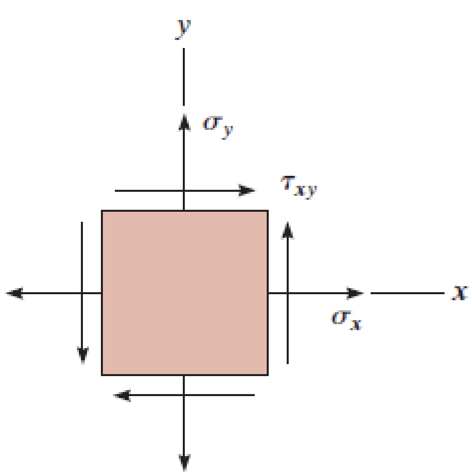
or simplifications of the loadings on a body in order that the stress produced in a structural member or mechanical element can be analyzed in a *single plane*. When this is the case, the material is said to be subjected to ***plane stress,*** Fig. *b*. *(the case when all the stress components in the Z-direction are zeros $\sigma_z = \tau_{yz} = \tau_{xz} = 0$.)*



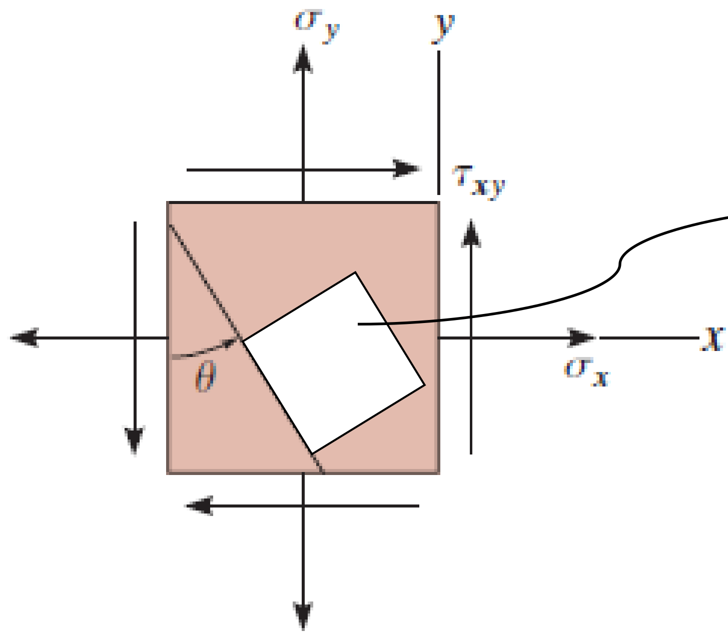
General state of stress
(a)

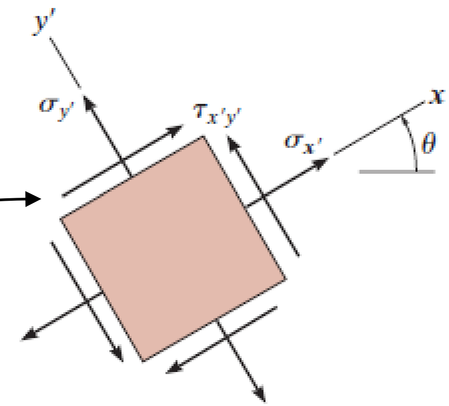
Plane stress
(b)

The plane stress state in Fig (b) above can represented as in Fig below

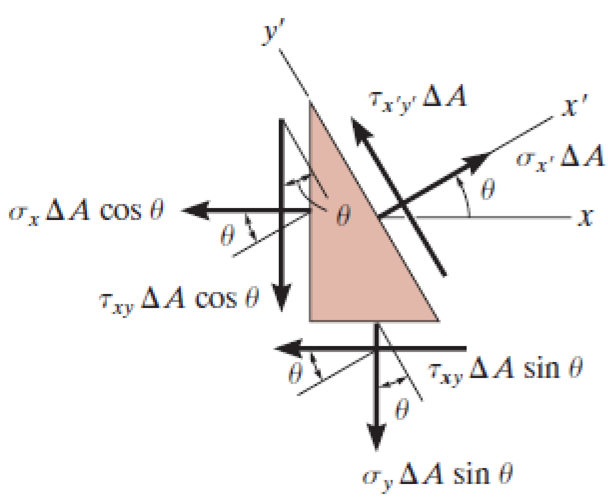


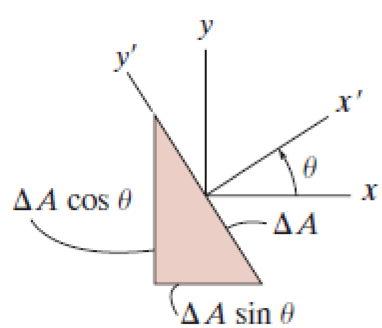
Imagine that we need to determine the stresses on an inclined square that is cut from the original square as in figure below. The inclined axes are x' and y' and the stresses in these directions are $\sigma_x{}'$, $\sigma_y{}'$ and $\tau_{x'y'}$





Assume that the area of each side of the inclined square is ΔA. The vertical and horizontal areas are as shown in the figure below to the right. The forces are shown in the figure top the left.





We need to convert stresses to forces to determine the equilibrium of the free body diagram. Stresses cannot be used in equilibrium calculations.

$$+\nearrow \Sigma F_{x'} = 0; \quad \sigma_{x'}\,\Delta A - (\tau_{xy}\,\Delta A \sin\theta)\cos\theta - (\sigma_y\,\Delta A \sin\theta)\sin\theta$$
$$- (\tau_{xy}\,\Delta A \cos\theta)\sin\theta - (\sigma_x\,\Delta A \cos\theta)\cos\theta = 0$$
$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$
$$+\nwarrow \Sigma F_{y'} = 0; \quad \tau_{x'y'}\,\Delta A + (\tau_{xy}\,\Delta A \sin\theta)\sin\theta - (\sigma_y\,\Delta A \sin\theta)\cos\theta$$
$$- (\tau_{xy}\,\Delta A \cos\theta)\cos\theta + (\sigma_x\,\Delta A \cos\theta)\sin\theta = 0$$
$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

These two equations may be simplified by using the trigonometric identities $\sin 2\theta = 2\sin\theta\cos\theta$, $\sin^2\theta = (1 - \cos 2\theta)/2$, and $\cos^2\theta = (1 + \cos 2\theta)/2$, in which case,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (9\text{–}1)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (9\text{–}2)$$

Since $\sigma_y{}'$is normal to $\sigma_x{}'$ then $\sigma_y{}'$ can be determined by substituting $\theta = \theta + 90°$ in the first equation to get

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (9\text{–}3)$$

Example

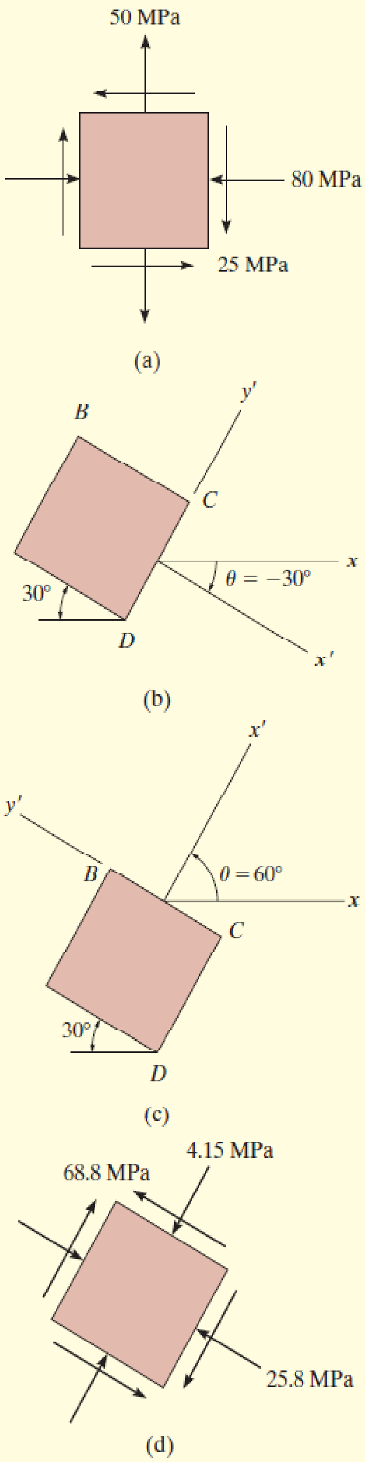


Fig. 9–7

The state of plane stress at a point is represented by the element shown in Fig. 9–7*a*. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.

SOLUTION

This problem was solved in Example 9.1 using basic principles. Here we will apply Eqs. 9–1 and 9–2. From the established sign convention, Fig. 9–5, it is seen that

$$\sigma_x = -80 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -25 \text{ MPa}$$

Plane *CD*. To obtain the stress components on plane *CD*, Fig. 9–7*b*, the positive x' axis is directed outward, perpendicular to *CD*, and the associated y' axis is directed along *CD*. The angle measured from the x to the x' axis is $\theta = -30°$ (clockwise). Applying Eqs. 9–1 and 9–2 yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(-30°) + (-25)\sin 2(-30°)$$

$$= -25.8 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-80 - 50}{2}\sin 2(-30°) + (-25)\cos 2(-30°)$$

$$= -68.8 \text{ MPa} \qquad \textit{Ans.}$$

The negative signs indicate that $\sigma_{x'}$ and $\tau_{x'y'}$ act in the negative x' and y' directions, respectively. The results are shown acting on the element in Fig. 9–7*d*.

Plane *BC*. In a similar manner, the stress components acting on face *BC*, Fig. 9–7*c*, are obtained using $\theta = 60°$. Applying Eqs. 9–1 and 9–2,* we get

$$\sigma_{x'} = \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(60°) + (-25)\sin 2(60°)$$

$$= -4.15 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{-80 - 50}{2}\sin 2(60°) + (-25)\cos 2(60°)$$

$$= 68.8 \text{ MPa} \qquad \textit{Ans.}$$

Here $\tau_{x'y'}$ has been calculated twice in order to provide a check. The negative sign for $\sigma_{x'}$ indicates that this stress acts in the negative x' direction, Fig. 9–7*c*. The results are shown on the element in Fig. 9–7*d*.

*Alternatively, we could apply Eq. 9–3 with $\theta = -30°$ rather than Eq. 9–1.

Principal stresses

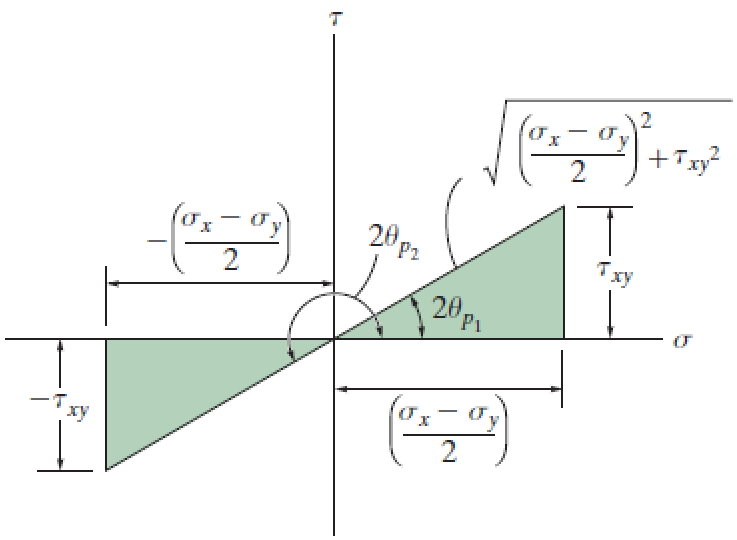
From the equation of $\sigma_x{}'$, $\sigma_y{}'$ and $\tau_{x'y'}$ that they depend of the orientation angle θ , so there must be a direction where the values of these stresses would be maximum or minimum. To find the maximum and minimum normal stress, we have to differentiate the equation of $\sigma_x{}'$ and set it to zero,

$$\frac{d\sigma_x{}'}{dx} = -\frac{\sigma_x - \sigma_y}{2}(2\sin(2\theta) + 2\tau_{xy}\cos(2\theta) = 0$$

Solving this equation, we obtain the orientation of $\theta = \theta_p$ of the planes of maximum and minimum normal stress.

$$\tan(2\theta_p) = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

From the from the shaded triangles shown in Figure below, the solution of the above equation has two roots $\theta_{p1} and\ \theta_{p2}$when both τ_{xy} and $(\sigma_x - \sigma_y)$ has same sign, positive or negative.



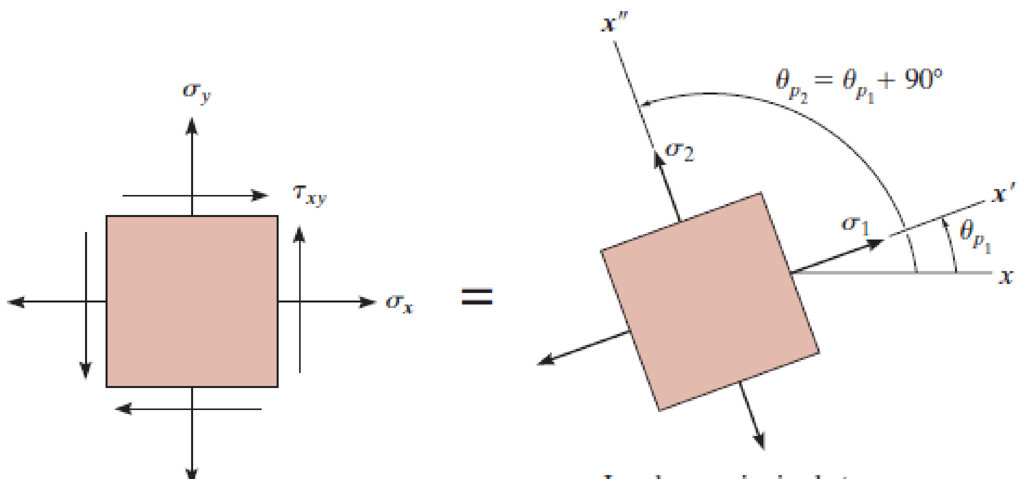
Substitute $\sin(2\theta_{p1})$, $\sin(2\theta_{p1})$, $cos(2\theta_{p2})$ *and* $cos(2\theta_{p2})$ from the figure above into Equation (9-1) to get maximum and minimum values of normal stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The cracks in this concrete beam were caused by tension stress, even though the beam was subjected to both an internal moment and shear. The stress-transformation equations can be used to predict the direction of the cracks, and the principal normal stresses that caused them.

$\sigma_1 and\ \sigma_2$ are the maximum σ_1 and minimum σ_2 in-plane stresses and also called the ***in-plane principal stresses.***

The planes on which principal stresses act are called ***principal planes. On these planes the shear stresses $\tau_{x'y'}$=0. In other words, no shear stresses on these planes.***

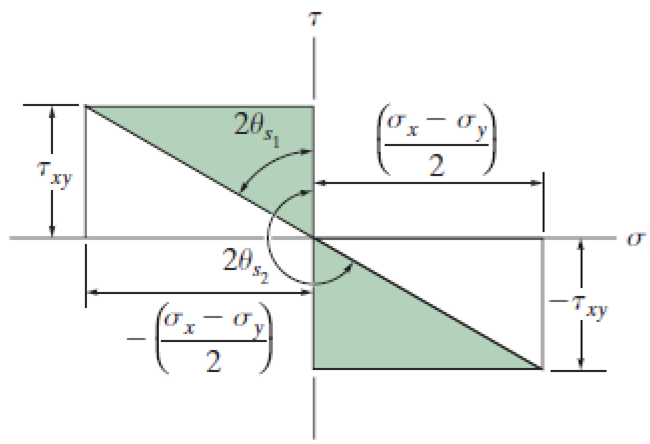


In-plane principal stresses

Maximum In-Plane Shear Stress.

The orientation of an element that is subjected to maximum shear stress on its sides can be determined by taking the derivative of Eq. 9–2 with respect to $\theta$and setting the result equal to zero. This gives

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$



The two roots of this equation θ_{s1} and θ_{s2} can be determined from the shaded triangles shown in the Figure above. By comparison with Eq of tan $(2\theta_p)$, of tan $(2\theta_s)$ is the negative reciprocal of tan $(2\theta_p)$ and so each root of $2\theta_s$ is 90 ° from $2\theta_p$ and the roots of θ_s and $\theta_p$is 45 ° apart.

Therefore, an element subjected to ***maximum shear stress will be 45° from the position of an element that is subjected to the principal stress***.

Substituting the sin $(2\theta_s)$ and cos$(2\theta_s)$ from the figure above into Eq (9-2) using either root of θ_s to get ***maximum in-plane shear stress***

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

Also substituting the sin $(2\theta_s)$ and cos$(2\theta_s)$ into Eq (9-1), we see that there is an associated normal stress that is called average normal stress

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Important Points

- The *principal stresses* represent the maximum and minimum normal stress at the point.
- When the state of stress is represented by the principal stresses, *no shear stress* will act on the element.
- The state of stress at the point can also be represented in terms of the *maximum in-plane shear stress*. In this case an *average normal stress* will also act on the element.
- The element representing the maximum in-plane shear stress with the associated average normal stresses is *oriented* 45° from the element representing the principal stresses.